

Family Business and the Quantity and Quality of Children
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Abstract

This study develops a household model of the demand for the quantity and quality of children in which family members can be employed in a family business. It is shown that the price of children will be lower because children has productive value in addition to consumption value. The price of children with respect to quantity will be negatively related to quality and vice versa. The implication is if the income elasticity of demand for quantity is less than for quality then an increase in income will lower the price of quantity relative to quality. This income induced substitution effect in favor of quantity against quality differs from the Becker and Lewis (1973) result on the interaction between quantity and quality of children. Micro survey data from the urban sector of Hong Kong are used to test the implications of the model. Estimation problems arising from the use of limited dependent variables are addressed by using well known econometric techniques. This study emphasises two distinct reasons why family business households have more children: (1) the complementarity between women's family business work and child care activities, and (2) the contribution of children's work in the family business.

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I. Introduction

The economic analysis of the demand for fertility makes an important distinction between price and income effects (see Becker, 1960). It is postulated that the price effect on the demand for children is negative, but the income effect is positive. This provides an explanation for the often observed negative correlation between fertility and income in cross-section and time-series data without having to make the onerous assumption that children are inferior commodities in the economist's unfortunate terminology. Given that men usually contribute more to family income than women and that women tend to spend more time caring for children than men, it was natural to associate husband's value of time with the income effect and wife's value of time with the price effect. Early studies of developed countries found strong evidence for negative price effects but inconclusive evidence for positive income effects.

The interaction between quantity and quality of children was emphasized by a number of authors to explain why observed income effects could be negative when in fact the true income effect may be positive. DeTray (1973) and Willis (1973) assumed that quantity and quality are close substitutes in the utility function or household production function. Becker and Lewis (1973) emphasized the point that the shadow price of children with respect to their number depends positively on their quality and vice versa. If the income elasticity of demand for quality is higher than quantity then an increase in income would induce a substitution effect in favor of quality against quantity. It is therefore possible for the number of children to decline with rising income. The evidence that years of schooling and the health of children tend to be negatively related to the

number of their siblings supports such an interpretation (see Blake 1981).

Comparative studies of developing countries have shown that the postulated negative relationship between women's employment and fertility to be weak and often absent (see United Nations, 1985). The usual explanation for this phenomenon is that the characteristics of certain types of market activity which women choose in developing countries are complementary with child care. An improvement in women's market opportunities in developing countries would encourage them to participate in these market activities which do not require them to significantly curtail their child care commitments. As a consequence, the negative price effect of a change in wife's wage rate on her fertility may be weaker in developing than in developed areas. Conceptually, market activities which have a high degree of complementarity with child care are characterized by flexibility in hours and place of work. In practice, the degree of complementarity is difficult to measure and the use of indirect proxies is a common practice. An important type of market activity often encountered in these studies is the home based family business. The pioneering study by Jaffe and Azumi (1960) emphasized that complementarity between wife's work and child care activities are enhanced in the family business household because home and workplace are the same. The cost of children to parents is thereby reduced. Most other empirical studies have found that women who work in the family business tend to have fertility levels that are higher than women in regular employment and sometimes as high as those of non-working women.¹

These extensions of the basic economic model of the demand for children were used to explain the anomalies of negative income effects and positive price effects observed in the data. However, if children are productive assets and work in the family business then family size will be larger if children are more productive working in the family business than elsewhere. This feature have been previously studied in the literature on farm households. Jaffe (1940), Gardner (1973),

Rosenzweig (1977), and others have shown that farm households have higher fertility rates than non-farm households. The observed positive relationship between family size and women's work in the family business cannot be attributed to complementarity effects alone. Notice that even if the wife does not work in the family business, such households will still have a larger family size.

If children work in the family business then their shadow price will also depend on the marginal value product of children in the family business. Since it is reasonable to assume that both the quantity and quality of children matters in the family business, the shadow price of children with respect to their quantity will depend on their quality and vice versa. Such results have a close affinity to the Becker and Lewis (1973) account of the interaction between the quantity and quality of children. The implications, however, will differ because in a family business model the price of children with respect to number may be negatively related to quality and vice versa, whereas in the Becker and Lewis model they are positively related. This difference implies that if the income elasticity of demand for quality is higher than quantity then an increase in income in a family business household could induce a substitution effect in favor of quantity against quality. If the substitution effect is dominant the observed effect of an increase in income would be to increase family size, which is in sharp contrast to the effects of income in non-family business households.

The family business has two distinct effects on the demand for the quantity and quality of children. Its presence alters the usual mechanisms by which income and price effects affect the demand for children. In order to distinguish between the economic contribution and complementarity interpretations, we examine theoretically and empirically the implications of a model of the demand for children in which household members can work in a family business. We take advantage of the richness of the various forms of market activity that exist in the urban sector of Hong Kong to examine how the demand for children is affected by the economic returns to different

types of market activity. We shall examine how changes in household income and wages rates of family members in different types of economic activity will affect parental choice between the quantity and quality of children on the one hand, and the type of economic activity they pursue on the other.

Section II describes in some detail the various types of market activity in Hong Kong and summarizes how fertility rates and child schooling vary with their choices. Section III develops a household production framework that models salient aspects of the family business and its relationship to the demand for children. Section IV defines the empirical variables and outlines a strategy for estimating the demand for children. Section V interprets the empirical results. The final section concludes with a brief summary of the major findings.

II. Data Description

Data from Hong Kong are used to examine the problem outlined in the previous section. The specific data set used is a one percent sample of the 1976 Hong Kong By-Census of Population. 1976 was World Fertility Survey year and there is considerable information on maternity history of women under age 50. For our analysis only women under 50 who are currently married and living with their husbands are included in the sample. The households are restricted to land based and non-farm families with husbands who are economically active. There are a total of 4,128 families in the sample. In 3,768 families the wife had experienced at least one child birth.

Economic activity is classified into four major categories. All those who worked for less than 15 hours during census week are classified as not working. By definition there are no such men in the sample since only those who are in the labor force are selected. Another form of market activity is work for a family business. Individuals who reported themselves as employers, self-employed, or family workers fall under this category. Although some employers may employ large numbers of

workers, the overwhelming proportion of these enterprises in Hong Kong is exceedingly small. The average number of persons engaged in all the registered private enterprises in Hong Kong is 12.7, however, 77 percent of the firms engage less than 10 persons with an average employment of 3.3 persons per firm. Family enterprises are interesting for two reasons. First, they permit women to engage in a type of market activity which is complementary with child care. Second, they often employ own children and so enhances the economic contribution of children. This second result occurs regardless of whether the wife works for a family business or not.

The wage employee is the most common type of market activity. These workers perform their jobs at the firm site. However, in Hong Kong a large fraction of the women in the labor force do not work at the firm site. They can take their work wherever they choose and are treated as a distinct category known as "outworkers". The kind of work they perform is quite diversified, but many are sewers, embroiderers, plastic product assemblers, and the like. Since they typically work at home we shall call them home workers. In our sample no men reported themselves as home workers, although there were small numbers of them in the population. This may reflect primarily our decision to include only prime age married men in the sample. For women, the major difference between working at home and working away from home is complementarity with child care activities, for example, working at home economizes on fixed commuting costs. Families with women home workers differ from those who operate a family business in that the latter may in addition also utilize child labor. It may be argued that women home workers may also utilize child labor to help perform some of her market tasks at home, in which case, families with women who work at home may not be very different from those with women who work for a family business. To the extent that women home workers perform unskilled market tasks with little need or opportunity to accumulate specific human capital, but women and children who work in a family business can and does accumulate firm

specific human capital, the difference between the two types of activities remains. The issue is to some extent empirical. An indirect test to see whether home workers and family business workers are identical is performed in section IV with negative results.

In Table 1 we cross tabulated husband's current economic activity against that of his wife's. The average number of children and an index of average child schooling level in each family are also given. The child schooling index is defined as the sum of the ratio of each child's schooling to the average schooling level attained by all children in the sample belonging to the same age and sex category. One can see a rich pattern of interaction between the economic activities of husbands and wives. Among family business men, 64 percent of their wives are not working, 18 percent are in the family business, 4 percent are home workers, and 14 percent are wage employees. Among husbands who are wage employees, 64 percent of their wives are not working, 2 percent are in a family business, 8 percent are home workers, and 26 percent are wage employees. These figures indicate that if a husband works in a family business, his wife is more likely to be similarly employed, but is less likely to be a wage employee working away from home, and is even less likely to be a home worker. Notice, however, that a significant number of women who work in a family business have husbands working in wage employment, and the converse is also true.

Men who are wage employees have fewer children, 2.94, than family business men, 3.63. Given wife's choice of economic activity, the average number of children per family is higher when the husband works for a family business. Non-working women have 3.18 children, which is not higher than the corresponding figures for family business women and home workers. Of even greater interest is that the number of children is greater among family business women, 4.19, than among home workers, 3.58. All these figures are significantly higher than that of wage employees, who have 2.45 children. This is consistent with the idea that unlike wage employment, family business

and home work are complementary with child care. The fact that the number of children among family business women are even higher than home workers, suggests that children's economic contributions in the family business may be an additional factor contributing to the demand for children. The results are unchanged even after controlling for husband's economic activity.

The figures for the child schooling index are more difficult to interpret since there is less variation across categories. In general, child schooling is lowest when the wife is a family business worker and second lowest when she is a home worker, but highest when she does not work in the labor force. There is an apparent negative correlation between the quantity and quality of children by wife's economic activity. There is little difference in child schooling attainment between households where husbands are wage employees and those where they work in the family business, despite the fact that the latter group have significantly higher earnings.

The summary evidence in Table 1 suggests that the number of children and their schooling attainment vary systematically with husband's and wife's economic activity. The distinctions we have made between wife's work and child care complementarity, on the one hand, and between family business activity and wage employment, on the other hand, are meaningful ones. Nevertheless, the figures on the average number of children per family and child schooling index by parents' economic activity are suggestive, and should be interpreted with caution since they are not standardized for other parental characteristics like age, schooling, and income.

Since the cross tabulations are of current economic activity status, they do not reveal how family members allocate their time among different activities over their life time. At any moment in time an individual may appear to be specialized in a particular economic activity status. But over the life cycle an individual may devote certain fractions of his total time to different activities, and life time specialization may be less pervasive. For example, setting up a family business requires financial

outlays which can be borrowed or saved only after one has accumulated work experience in the labor market, unless, of course, one inherits a business. Similarly, home work is valuable to women who have a high value of home time, say during the childbearing and child rearing years, and who have low market productivity. These intertemporal aspects are clearly important, however, in the absence of relevant data on economic activity history for the individual, we can only focus our analysis on outcomes at a moment in time.

III. Theoretical Framework

In this section a simple one period model of household production and the demand for child services is outlined. The model is a variation of Becker and Lewis (1973), Rosenzweig (1977), and Wong (1988). Only the major implications of the model are developed. It is assumed that all decisions are made jointly by the parents, who obtain utility from three household commodities described by the utility function

$$(1) \quad U = U(N, Q, Z)$$

where N is the number of children, Q is the quality of children, and Z is the flow of services from a residual composite commodity.

Children are assumed to be more productive when they work in the family business than when they are employed elsewhere. This is because perfect substitutes may not exist for own children and specific human capital may be embodied in children. To simplify the exposition, we shall only include **net** services contributed by children in the family business production function. The implied profit function is given by

$$(2) \quad \Pi = P[F(N, Q) + G(e_f T_{fb}, e_m T_{mb}, K)] - rK - W_f T_{fb} - W_m T_{mb}$$

where $F(\cdot)$ and $G(\cdot)$ are twice differentiable decreasing returns to scale production functions with $F_i, G_i > 0$ and $F_{ii}, G_{ii} < 0$. The assumption that the net services of children and the other factors are

separable greatly simplifies some of the rather involved manipulations without sacrificing the major insights of the model. T_{fb} and T_{mb} are time spent by wife and husband in the family business, e_f and e_m are their efficiency coefficients, W_f and W_m (W_f and W_m) are the market (shadow) prices of time for wife and husband, K is hired physical or human capital services, r is the rental price of capital services, and P is the price per unit of output.

The production functions for commodities N , Q , and Z are assumed to be linear homogeneous and are given by

$$(3) \quad J = J(X_j, T_{fj}, T_{mj}; e_{fj}, e_{mj}) \quad (j, J = N, Q, Z)$$

where purchased goods X_j and time of parents T_{fj} and T_{mj} are inputs, and e_{fj} and e_{mj} are efficiency coefficients. Although the husband may spend time in household production, but to focus attention on the crucial features of the model we shall assume that he does not do so. The husband therefore may only allocate his time between the family business, T_{mb} , and wage employment, T_{me} . In equilibrium, his marginal value product in the family business, $Pf_{T_{mb}}$, equals his wage rate, W_m , in wage employment. His time constraint is given by

$$(4) \quad T_m = T_{mb} + T_{me}$$

In addition to household production, the wife also works in the family business, T_{fb} , and in wage employment, T_{fe} . In equilibrium, her marginal value product in the family business, $Pf_{T_{fb}}$, equals her wage rate, W_f , in wage employment, which also determines her value of time. Her time constraint is

$$(5) \quad T_f = T_{fn} + T_{fq} + T_{fz} + T_{fb} + T_{fe}$$

The family's goods constraint is given by the non-linear expression

$$(6) \quad V + W_f T_{fe} + W_m T_{me} + \Pi = p_n x_n N + p_q x_q Q + p_z x_z Z$$

where V is non-earnings income, p_n , p_q and p_z are price indices of the composite goods used in the household production of N , Q and Z ; and x_n , x_q and x_z are the marginal (=average) composite goods

coefficients of N, Q and Z. The constraints (4), (5), and (6) can be combined under the assumption that parent's value of time are W_f and W_m to yield

$$(7) \quad I = V + W_f T_f + W_m T_m + \Pi = \pi_n N + \pi_q Q + \pi_z Z$$

where I is full income and π_j is the constant marginal shadow cost of commodity **j**. The budget constraint (7) is non-linear because of the family business profit function. In the Becker and Lewis (1973) formulation, a non-linear budget constraint, is a result of the inherent complementarity between the quantity and quality of children. An increase in Q raises the price of N, and vice versa.

If the husband does not spend any time in household production then the constant marginal shadow cost function is given by

$$(8) \quad \pi_j = p_j x_j + W_f t_{fj}$$

where t_{fj} is the marginal time coefficient of wife's time in the production of commodity **j**. However, π_j ($j = N, Q$) is not the true shadow price of children with respect to N or Q in either consumption or production. As we shall see, the prices of children in consumption and in production are not constant. To distinguish these two price concepts from π_j , we have deliberately chosen to refer to it as the shadow cost rather than the shadow price of children.

The optimal consumption of N, Q and Z and the optimal utilization of husband's and wife's time and of capital services are obtained by maximizing (1) subject to (7). Assuming that an interior solution exists, the first-order conditions for an optimum are

$$(9) \quad U_j + \mu(PF_j - \pi_j) = 0 \quad (j = N, Q)$$

$$(10) \quad U_z - \mu\pi_z = 0$$

$$(11) \quad PG_{T_{ib}} - W_i = 0 \quad (i = f, m)$$

$$(12) \quad PG_k - r = 0$$

$$(13) \quad V + W_f T_f + W_m T_m + \Pi - \pi_n N - \pi_q Q - \pi_z Z = 0$$

where μ is the marginal utility of full income.

Equations (11) - (12) are the first-order profit maximizing conditions. Husband's and wife's time and capital services are employed such that the contribution to income of the last unit of each factor equals its market price. An increase in W_m (W_f) will encourage the husband (wife) to allocate more time to wage employment. On the other hand, an increase in the efficiency of time in family business production will induce husband or wife to allocate more time to the family business. It is conceivable that equation (11) may not hold under some circumstances. A sufficiently high W_m (W_f) may lead the husband (wife) to specialize in wage employment, and the converse can lead the husband (wife) to withdraw from wage employment completely. At any moment, it is quite common for household members to specialize in an economic activity. The majority of the men specialize in wage employment and a smaller fraction in the family business. The majority of the women do not work, and for those who do, they usually specialize either in wage employment or in the family business. There are numerous ways in which a husband's choice of economic activity can be matched to that of his wife. It is sometimes useful to recognize the possibility of specialization in an economic activity when we ascertain the response of the demand for children to changes in the parameters of the model.

There are several important features in our model. First, children appear in both the utility and production functions. Consequently, the desired levels of consumption of children with respect to N and Q exceed the amounts that would have been demanded had children provided only production or consumption value. Rewriting expression (9) yields (14),

$$(14) \quad \pi_j^* = U_j / \mu = \pi_j - PF_j \quad (j = N, Q)$$

which shows that the optimal level of N or Q is obtained when its marginal revenue is less than its

cost by the income value of its marginal utility. The true shadow price of children, π_j^* , contains a child productivity component, which equals the marginal value product of children in the family business. One may think of the productivity component as subsidizing the consumption component, and vice versa.

Second, although the marginal shadow costs of children are constant, the shadow price of children, π_j^* , depends on both N and Q because of the productivity component in price. An interesting issue is the sign of F_{nq} . We assume that N and Q are complements in the family business production function, the marginal value product of N will depend positively on Q , and vice versa. Hence, the shadow price of children with respect to N will be negatively related to Q , and vice versa. This feature of our model differs from the Becker and Lewis (1973) model, where the price of N is positively associated with Q , and vice versa. The implications for income and substitution effects with respect to both N and Q in our model are potentially different from theirs.

Third, when the wife does not spend any time in wage employment other considerations are relevant. Her time will be allocated between household production and the family business. Notice that her value of time, W_f , is endogenous even though she is engaged in market activities. The marginal shadow costs of children, π_n and π_q , also become endogenously determined. Some of the implications will be briefly explored later.

1. Income effects. - The effect of an increase in non-earnings income V in a household without a family business is given by $(V/I)n_j$, where n_j is the income elasticity of demand for j . In a family business household the full income budget constraint becomes non-linear in N and Q . The effect of an increase in non-earnings income V on the demand for children has both a substitution and a pure income component (see Edlefsen 1981).

Assume that N and Q are normal commodities. The observed income effects depend on two

opposing price level effects which determine real household income. First, the dual role of children in a family business household implies that the true price of children is reduced. This effect alone implies that the observed income effects will be higher than their true effects. Income is increased by the ratio $k = (I-R)/R$, which depends on the importance of the productivity components of children in prices. R is the expenditure on N , Q , and Z defined as $R = \pi_n N + \pi_q Q + \pi_z Z$. Clearly, $I-R = PF_q Q + PF_n N$. Second, as N and Q rise the price of children will also increase because their marginal productivity in the family business will fall due to diminishing marginal productivity. This would reduce the increase in real income. The net effect would depend on the income elasticities of demand for N and Q .

The exact conditions are derived in Appendix I. From equation (A8) in Appendix I we know that

$$E(R/\pi^*) - E(I/\pi) = (k - k_n s_n - k_q s_q) E(I/\pi),$$

where E is the logarithmic differential operator, π^* and π are price indices for shadow costs and prices defined by equations (A4) and (A5), $k_i = \pi_i i / R$ is the expenditure share of N , Q or Z with respect to income R , and $s_i = PF_i / \pi_i$ is the ratio of the marginal value product of i to its corresponding shadow price. This implies that an increase in real income measured in terms of I would lead to a larger increase in real income as measured by R , if the weighted sum of the income elasticities of demand for N and Q was less than k , where the weights are $k_n s_n$ and $k_q s_q$, respectively.

Casual evidence suggest that this might be the case because the income elasticity of demand for N is found to be low if not negative and that for Q is not high.

An exogenous increase in non-earnings income, holding constant husband's and wife's value of time, will increase the demand for N and Q . This gives rise to substitution effects due to relative changes between the shadow prices of children with respect to N and Q . If the income elasticity of

demand for Q is significantly larger than that for N, as is often presumed, then it is quite likely that PF_n will rise relative to PF_q . This occurs both because diminishing marginal productivity reduces F_q more than F_n and because complementarity between N and Q raises F_n more than F_q . It is plausible to presume that the shadow price of N will thereby decline relative to the price of Q in response to an increase in income. The exact conditions for this to be the case is given in equation (A9) of Appendix I. Notice that the Becker and Lewis (1973) model predicts an opposite outcome.

The most important points to note are that the observed income effects for both N and Q are likely to be larger in family business households, and that the substitution effects work in favor of N but against Q. These effects may be important in many agricultural societies and developing countries where family businesses are an important form of economic organization. If the substitution effects are dominant, the observed income responses of N may even be larger than Q. A decline in the importance of family businesses as an economic activity in the transition from a traditional to a modern society leads not only to a decline in the demand for children, but also to a rapid shift away from N into Q. Our results are consistent with those of Becker and Lewis (1973), but recognize the relevance of the family business as an alternative vehicle for these effects. On the other hand, an increase in income that is not accompanied by a shift into wage employment could actually increase the demand for N relative to Q.

2. Husband's wage rate. - Since husband's time is not an input into the production of children, the effects of an increase in his wage rate is analogous to pure income effects. There are three analytical possibilities. When the husband specializes in wage employment and the household does not have a family business, the effect of an increase in his wage rate on the demand for N or Q is similar to an increase in income and the elasticity effect is $(W_m T_m / I) n_j$. When the husband spends time in wage employment and the household has a family business, an increase in his wage rate increases

household income and shifts his time out of the family business. By assumption husband's time and children's time are separable in the family business production function. Nevertheless the prices of N and Q will still be affected by changes in his wage rate because the income effects generate a substitution effect in favor of N against Q.

When the husband works solely in the family business, an increase in his wage rate has no effect on the demand for children unless he shifts some time into wage employment as a consequence. His value of time equals the marginal value product of his time in the family business and is greater than his wage rate ($\bar{W}_m = PG_{T_{mb}} > W_m$). These considerations imply that an increase in husband's wage rate lowers his probability of working in the family business, which tends to reduce the demand for N and Q because children are more valuable in households with a family business. As households withdraw from the family business, the income induced substitution effect against Q in favor of N disappears. One might therefore observe a negative effect on the demand for N but not for Q.

3. Wife's wage rate. - When the wife does not engage in wage employment, an increase in her wage rate will have no effect on the demand for N and Q provided that she does not shift time into wage employment as a result. If the wife spends some time initially in wage employment then a compensated increase in her wage rate increases the marginal shadow costs for all three commodities N, Q and Z according to

$$E\pi_i = a_i EW_f; \quad a_i = (W_f t_{fi}) / \pi_i; \quad (i = N, Q, Z),$$

where a_i is the relative cost share of wife's time in the production of commodity i and is a measure of the time intensity of commodity i in cost terms. The appropriate measure of time intensity in price terms is given by $a_i^* = (W_f t_{fi}) / \pi_i^*$. For Z the two measures of time intensity are equal, but for N and Q we have $a_i < a_i^*$. So when the time intensities in cost terms are equal for all commodities, the time

intensities for N and Q in price terms will be greater than Z. An income compensated increase in wife's wage rate will therefore decrease the demand for N and Q relative to Z. The effect will be strengthened if the usual presumption that Z is less cost time intensive than N and Q is assumed.

The marginal value products of children in the family business with respect to N and Q will not remain constant when the demand for N and Q declines relative to Z. The relative shadow price between N and Q may therefore change even though the relative marginal shadow cost between N and Q remains constant. If the price of N rises relative to Q then the demand for N will fall relative to Q. Since $\pi_n/\pi_q = (s_q/s_n)(PF_n/PF_q)$ this requires that the marginal value product of children with respect to N must rise relative to that with respect to Q. In Appendix I we show that under rather plausible assumptions the income compensated demand for N would fall relative to Q in response to an equal increase in the shadow costs of all commodities. The effect would be strengthened if N is more time intensive to produce than Q is in cost terms.

4. Wife does not engage in wage employment. - When a woman spends all her time at home or in the family business, her value of time W_f is endogenously determined. There are two important consequences. The effect of an increase in non-earnings income V on the demand for children is ambiguous because the value of wife's time in household production also rises. This gives rise to a negative substitution effect away from children but a positive income effect.

5. Family business productivity effects. - An increase in e_m or e_f raises the efficiency of husband's or wife's time in the family business. This increases the scale of the family business. Real income in the household will rise and the optimal levels of N and Q that are demanded will also rise. There will also be an income induced substitution effect in favor of N against Q as the price of N falls relative to Q. One would therefore observe stronger positive effects for N, but weaker effects for Q. Since wife's time is an input into the production of N and Q, an increase in e_f may increase the

effective amount of time she can spend on child care and other household activities.

The above model implies a set of demand equations for N and Q for each combination of husband's and wife's choice of economic activity. For example, if both husband and wife engage in wage employment and in a family business then we have the following set of demand equations for children:

$$(15) \quad \mathbf{j} = \mathbf{j}(W_m, W_f, b_m, b_f, V, r) \quad \mathbf{j} = (N, Q)$$

If husband and wife do not take up in wage employment then W_m and W_f will not enter as arguments in equation (15). Similarly, if husband and wife do not work in a family business then b_m and b_f will not enter as arguments in equation (15), assuming that parents' productivity in the family business are contingent on having spent time working there. If we assume that a wife always spends some time in household production, then a total of 12 possible such demand functions for children can be specified depending upon parents' choice of economic activity. In the previous section we also indicated that family business workers and home workers may be considered as distinct categories on empirical grounds. The total number of possible choice combinations will then be 24.

IV. Empirical Methodology and Specification of Variables

Our aim here is to devise a tractable strategy for estimating the change in the desired levels of children as a function of the explanatory variables in the model. In principle, we have to specify an estimating equation conditional on each choice of parental economic activity. Since the theory is based on a static one period model, we need information on how parents allocate their total endowment of time over their lifetime. We must also have lifetime measures of the variables used in the analysis. Such a strategy cannot be feasible with data from a single cross section. An alternative is to adopt the simplifying assumption that both parents spend some time in each activity. The appropriate set of equations for estimating the demand for the quantity and quality of children for all

households would be given by expression (15). In section II above we indicated that although an individual is reported to be specialized in a particular activity at any given moment, however, he or she may engage in different economic activities over the life cycle. Lifetime specialization in a particular activity may be much less pervasive than is presupposed in cross section data. But in the absence of adequate data to verify the appropriateness of this assumption, caution should be exercised when interpreting the empirical results.

To estimate a lifetime model of the demand for children, we need measures of completed fertility of women, and of husband's and wife's lifetime productivity in household production, wage employment, home work, and the family business. Direct measures of these variables are not available and have to be constructed.

1. Number of children. - Current fertility among young women often differs significantly from their completed levels. However, the reduction in sample size is unacceptable, especially for women home workers, if we were to restrict it to include only those women who are over 35 years of age. An alternative solution is to adopt a procedure used by Anderson (1983) based on the duration ratio concept developed by Boulier and Rosenzweig (1978). This is defined as the ratio of the number of children ever born to the number of children a woman could have if she had reproduced according to a natural fertility schedule. It is an attempt to standardize for cumulative fertility due to age. A plot of cumulative fertility against women's age in Hong Kong reveals a S-shaped pattern of growth. The procedure used here is to estimate the demand for the number of children constrained by a non-linear biological supply function, with fertility beginning at the age of menarche. The estimated function is

$$N^* = N\{1 - \exp[-d_1 (AGE - d_0)]\}^3$$

where N^* is the current number of births, N is the cumulative supply of births at the end of childbearing, AGE is the current age of the wife, $d_0 = 15$ is assumed to be the age of menarche, and

d_1 is a parameter to be estimated. Rewriting in logarithms yields

$$\ln N^* = \ln N + 3 \{1 - \exp[-d_1 (AGE - d_0)]\}$$

where $\ln N$ depends on a vector of explanatory variables. The above function can be estimated using non-linear least squares.

2. Child schooling. - Years of schooling is used as an indicator of child quality. An index of the average schooling level attained by children in the household is computed. It is defined as the sum of the ratio of each child's schooling to the average schooling level attained by all children of the same age and sex group in the sample. The value is then divided by the number of children in the household. Only children above the age of 4 is used to construct the index. Note also that the index can be defined only for families with at least one child.

3. Market productivity. - The construction of variables for measuring market productivity in wage employment (W_m, W_f), and in family business (e_m, e_f) is somewhat indirect. Data are available only for the current market activity of husband and wife. There is no information on how their time is or will be allocated among different uses over their lifetime. Consequently, market productivity can only be observed for individuals in their chosen market activity. Measures of potential market productivity under alternative choice of market activity status has to be constructed. Heckman (1979) has shown that direct estimates of market productivity functions using samples of those in a particular market activity may be biased due to self-selection. To obtain consistent estimates he proposed a two-stage method. First, we estimate separate reduced-form choice models for husband's and wife's probability of selecting a particular activity. Next, we estimate the coefficients of the market productivity functions conditional on the probability of selecting a particular activity.² In the actual estimation we use a dichotomous logit model to describe men's choice, and a polychotomous logit model to describe women's choice. The properties of multinomial logit models are discussed in

McFadden (1982). Both models specify husband's and wife's choice of economic activity as functions of the same set of explanatory variables.

In order to test whether home work and family business work are identical alternatives for women, we estimate both a three-way choice model where these two choices are combined into a single category, and a four-way choice model where they are treated as separate categories. The likelihood ratio test of the hypothesis that the two parameter vectors affecting entry into home work and family business work are equal shows that the null hypothesis should be rejected at any conventional level of significance.³ The four-way choice model is adopted in the rest of the analysis in this study. The maximum likelihood estimates of the reduced-form multinomial logit models for husband's and wife's choice of economic activity are given in Appendix 2.⁴

From these reduced-form logit estimates we can construct a selectivity bias correction variable, $\hat{\lambda}$, and use it as an additional regressor in the estimation of market productivity functions for individuals with the same market activity. This variable was derived by Lee (1983) for use in polychotomous choice models. The dependent variables used in the estimation of the market productivity functions are the log value of monthly earnings. The hourly wage rate is preferable but was not reported for all individuals. Monthly earnings are less desirable estimates of market productivity because they include variations in hours worked, and as a result are jointly determined with the level of children. The estimates of husband's and wife's log earnings functions conditional on the chosen economic activity using least squares methods are given in Appendix 3. These estimates are used to impute consistent values of market productivity (log earnings) for each individual in every choice of potential or actual market activity.⁵ These imputed measures are less affected by transitory events. The influence of life cycle effects is removed by imputing the market productivity of wives at a fixed age, say 40. Husband's imputed market productivity is set at 40 plus the actual husband-wife age

differential.

4. Non-market productivity. - Both husband's and wife's schooling are often found to have independent effects on raising non-market productivity, including the production of children. One might, therefore, wish to include them as additional regressors in the demand for children equations to proxy for variations in non-market productivity.

5. Joint estimation. - Table 2 lists those variables used in the analysis (including those reported only in the Appendices) with their summary statistics. A system of demand equations for fertility and child schooling can be estimated as functions of the above variables which measure the productivities of husband and wife in different market activities and in household production. A generalized least squares procedure is adopted to obtain efficient estimates of the coefficients. These joint estimates are non-linear in the fertility equation and linear in the child schooling equations.

V. Empirical Results

This section interprets some of the major findings of the empirical work. Since the child schooling variable is not defined for families with no children, the system of demand equations for fertility and child schooling is estimated using the sample of families with at least one child. Table 3 gives joint estimates for these equations. For comparison purposes we also estimated the fertility equation using the larger sample. The parameter estimates in the two samples show that the coefficients of the non-linear fertility equations have identical sign patterns and are often of similar magnitudes.

An increase in husband's productivity in the family business, measured by predicted log earnings, increases both fertility and child schooling. The effect is, however, stronger and more significant on fertility than it is on child schooling. If the income elasticity of demand for Q is greater than N, the income effect would cause Q to increase more than N. This suggests that there is

probably a significant substitution effect against Q in favor of N due to an induced decline in the price of N relative to the price of Q.

An increase in husband's predicted log earnings in wage employment has a significant negative effect on fertility but has no effect on child schooling. Most other studies have found negative fertility effects and positive child schooling effects for which the Becker and Lewis (1973) model was designed to explain. The absence of a positive child schooling effect is somewhat unusual but can be explained in our model. One interpretation of our model is to assume that husbands always specialize in one type of economic activity. An increase in husband's predicted log earnings in wage employment increases the probability of specializing in that activity. Since children are more valuable in family business households, the demand for N and Q will be less as husband's wage rate rises. The negative effect may be stronger for N than Q because the income induced substitution effect among family business households in favor of N disappears when the husband withdraws completely from the family business and specialize in wage employment.

However, if the husband spends time in both wage employment and the family business then one would expect the demand for N to increase. The effect on Q may be negative if the induced substitution effect dominates the income effect. Under this latter interpretation, our model cannot explain the observed findings and needs to bring in some features of the Becker and Lewis (1973) model.

An increase in wife's predicted log earnings in wage employment has a negative effect on fertility and child schooling. Both effects are statistically significant. The point estimates appear to be smaller for child schooling as expected. But this may be because child schooling variable is poorly measured as compared to the fertility variable. Our findings are consistent with the prediction that the uncompensated response of N to an increase in wife's wage rate is more likely to be negative

than Q . The income effects associated with an increase in wife's wage rate are probably of minor importance being dominated by the substitution effects.

An increase in wife's productivity in the family business, proxied by predicted log earnings in the family business, increases both fertility and child schooling. However, only the fertility effect is significant. The results are again consistent with the prediction that substitution effects in favor of N against Q can be important whenever there is an income induced increase in the demand for children.

The mechanisms are basically similar to those due to an increase in husband's productivity in the family business and so are the observed effects.

It is worth noting that the negative association between wife's work in the family business and fertility have often been interpreted as a result of complementarity between these two activities. Our explanation does not make this assumption although we believe that complementarity may also be part of the reason. However, unless we have direct measures or proxies to identify these effects separately, it is not possible to distinguish between them. The insignificance of the child schooling response to an increase in wife's productivity in the family business would be more difficult to explain if only the complementarity effect mattered. One would then presumably have to argue that wife's time in the family business is detrimental to the production of child quality.

A better proxy for capturing child care complementarity effects is probably home work. An increase in wife's predicted log earnings in home work is found to have significant positive effects on both fertility and child schooling. The shift of wife's time into an activity which is complementary with child care is conducive to the production of children with respect to both N and Q . The finding that the child schooling effect is positive distinguishes home work from family business work. It appears that home work is not detrimental to increasing child schooling levels.

Both husband's and wife's schooling reduce significantly the demand for fertility but increase it

significantly for child schooling. Furthermore, wife's schooling effects are found to be larger than husband's schooling effects. These results have been duplicated in many studies. Since husband's and wife's market productivity in wage employment and in the family business have been held constant, schooling is a proxy for efficiency in household production. An increase in efficiency would reduce the shadow cost of children, especially with respect to Q. This translates into a relative fall in the shadow price of Q and induces an increase in Q, which in turn induces an increase in the shadow price of N through a Becker and Lewis (1973) mechanism. In our model, this effect may be partially offsetted by a substitution effect in favor of N due to the productivity component in the price of children.

Parents' schooling is highly correlated with measures of market productivity and this may affect some of our estimations. To test whether correlations between schooling and the predicted log earnings variables might undermine the robustness of our findings, we reestimated the regressions without husband's and wife's schooling variables. The unreported results indicate that none of the included coefficients are significantly altered.⁵

VI. Conclusion

In this paper we emphasize two aspects of the demand for children in family business households. First, since children can be employed in the family business there is a greater demand for them. The demand is biased in favor of the quantity and not quality of children due to induced substitution effects. Consequently, in contrast to Becker and Lewis model, when income rises the observed response in the quantity of children dominates the response of quality. This means that when our model is combined with the Becker and Lewis model, increases in income which shifts economic activity away from the family business could result in very rapid declines in the quantity of children and advances in their quality.

Second, if the wife works in a family business or as a home worker, the demand for children will also increase. The key issue here is to distinguish between complementarity effects and the effect of children's economic contribution in the family business. We find evidence for both effects. Distinguishing between different types of economic activities is an important aspect of testing for these two effects. We assumed that home work is a better proxy for capturing complementarity effects than family business work. We found that women with a higher predicted earnings in home work tend to have higher fertility rates and child schooling levels, but those with a higher predicted earnings in family business work only experience higher fertility rates. This differential response is consistent with the idea that in family business households there is an induced substitution effect in favor of the quantity of children.

Previous studies of the demand for children tend to put emphasis on wife's market activity and its complementarity with child care. Although there are extensive discussions of the importance of children's contribution to family income, the central role of husband's market activity as a key determinant of the quantity and quality of children have not always been systematically incorporated into the analysis. Furthermore, the respective roles of these two types of effects have often been confounded because of working with inadequate data. Progress has been achieved to some extent in this paper by making use of the richness of the dataset on different types of market activity in Hong Kong.

Footnotes

- * This research was completed while the author was a Visiting Scholar at the Hoover Institution, Stanford University. An earlier version was presented at the 1993 Far Eastern Econometric Society Meeting of the Econometric Society, Taipei, Taiwan. The usual disclaimer applies.
1. The common practice in many studies is to identify those market activities with a high degree of complementarity on the basis of work characteristics. Work in the informal sector was assumed to have a high degree of complementarity. The early study by Jaffe and Azumi (1960) defined the informal sector in Japan to include the self-employed and family business workers. Subsequent work on Japan by Hill (1983) continued to use this definition. McCabe and Rosenzweig (1976), Blau (1984), and others defined formal and informal sector work by occupation and industry classifications. Smith (1981) used a much greater variety of definitions, including part-time work and work at home (also known as the putting-out system). The numerous definitions used in these studies demonstrate the rich variety of market activities that exist in developing areas. These definitions may overlap each other to some degree. For example, cottage industries are sometimes found mainly in certain occupations and industries, and they may be reported as part-time work or work at home. The extent of overlap may vary from one place to another.
 2. We recognize that husband's choice of an economic activity may not be independent of wife's choice, and vice versa. Therefore, the above specification may be inappropriate, however, given that husbands have two choices and wives have four choices, the total number of observable combinations will be eight, with each cell in Table 1 representing a single outcome. Unfortunately this results in very small cell sizes for certain outcomes, so that it may be difficult to obtain reliable estimates of the joint probabilities. The large number of choice categories is also a troublesome feature when specifying an appropriate model for estimation purposes. The flexible multinomial probit model will be computationally far too cumbersome, but the simpler multinomial logit model contains the independence from irrelevant alternatives property, which is unlikely to be correct in this instance. Experiments with nested logit models had problems of converging. In view of the difficulties in getting correct estimates of the joint probabilities of husband's and wife's choice of activity status, we decided on the approach described in the text.
 3. The test statistic for the reduced-form model is 42.0. The chi-square values at a .01 level of significance is 26.2 with 12 degrees of freedom.
 4. The interpretation of the estimated coefficients are omitted as they are not essential to the ideas in this study. A copy of discussion notes can be obtained from the author upon request.
 5. These additional regressions may be obtained from the author upon request.

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Appendix 1

The total expenditure on N, Q and Z evaluated at their true shadow prices is given by

$$(A1) \quad R = \pi_n^* N + \pi_q^* Q + \pi_z^* Z,$$

which is related to observed income, I, as

$$(A2) \quad R = I - P F_q Q - P F_n N = I/(1+k),$$

where $k = P(F_n N + F_q Q)/R$ is the share of children's contribution to household income. (A2) can be expressed in logarithmic differential form as

$$(A3) \quad E R = (1+k) E I - k [E P + b_n E N + b_q E Q],$$

where E is logarithmic differential operator and b_i is the partial elasticity of output with respect to i , $E F / E i$, in the family business production function. If $F = F(NQ)$ then $b_n = b_q$. We let $E P = 0$.

We now define two household price indices π^* and π in logarithmic differential form as

$$(A4) \quad E \pi^* = k_n^* E \pi_n^* + k_q^* E \pi_q^* + k_z^* E \pi_z^*,$$

$$(A5) \quad E \pi = k_n E \pi_n + k_q E \pi_q + k_z E \pi_z \\ = [k_n^*(1+s_n) E \pi_n^* + k_q^*(1+s_q) E \pi_q^* + k_z^* E \pi_z^*] / (1+k),$$

where $k_i^* = \pi_i^* i / R$ is the expenditure share of N, Q or Z with respect to income R, $k_i = \pi_i i / I$ is the expenditure share of N, Q or Z with respect to income I, and $s_i = P F_i / \pi_i^*$ is the ratio of the marginal value product of N or Q to its corresponding shadow price.

Equation (14) can be expressed in logarithmic differential form as

$$(A6) \quad E \pi_i^* = (1+s_i) E \pi_i - [E P + s_i b_j E j - s_i (1-b_i) E i],$$

$$(i \neq j = N, Q).$$

Substitute (A6) into (A4) to obtain (A7) noting that $E P = 0$.

$$(A7) \quad E \pi^* = E \pi - k_n^* [s_n b_q E Q - s_n (1-b_n) E N] \\ - k_q^* [s_q b_n E N - s_q (1-b_q) E Q].$$

Subtract (A7) from (A4) to obtain (A8) noting that $k = k_n s_n + k_q s_q$.

$$(A8) \quad E(R/\pi^*) = (1+k)E(I/\pi) - k_n s_n EN - k_q s_q EQ.$$

From (A8) we can see that an increase in real income measured in terms of I would lead to a larger increase in real income as measured by R , if the weighted sum of the income elasticities of demand for N and Q was less than k , where the weights are $k_n s_n$ and $k_q s_q$, respectively. Casual evidence suggest that this might be the case because the income elasticity of N is found to be low if not negative and that of Q is not high.

An exogenous increase in income holding constant the π 's will increase the demand for N and Q . This produces substitution effects due to changes in the shadow price of N relative to the price of Q . Using equation (A6) we obtain

$$(A9) \quad E\pi_n - E\pi_q = [s_q b_n + (1-b_n)s_n]EN - [s_n b_q + (1-b_q)s_q]EQ.$$

Note that we have set $EP = E\pi_n = E\pi_q = 0$. If the income elasticity of demand for Q is significantly larger than that for N then it is quite likely that the shadow price of N will decline relative to the price of Q in response to an increase in income. If $b_n = b_q$ then a sufficient condition for the price of Q to rise relative to N would be for the income elasticity of demand for Q to be less than N .

The observed income and substitution effects can be formally derived by making use of the well known propositions that

$$(A10) \quad EN = \eta_n E(R/\pi^*) + k_z \sigma_{nz} E\pi_z - (1-k_n) \sigma_n E\pi_n + k_q \sigma_{nq} E\pi_q,$$

$$EQ = \eta_q E(R/\pi^*) + k_z \sigma_{qz} E\pi_z + k_q \sigma_{nq} E\pi_n + (1-k_q) \sigma_q E\pi_q,$$

$$(1-k_n) \sigma_n = k_z \sigma_{nz} + k_q \sigma_{nq}; \quad (1-k_q) \sigma_q = k_z \sigma_{qz} + k_n \sigma_{nq},$$

where the σ_{ij} 's are the Allen partial elasticities of substitution in the utility function. Notice that σ_n is the average elasticity of substitution of N against Z and Q and that σ_q is the similar elasticity for Q against Z and N . The observed income elasticities for N and Q can be obtained by changing income,

I, but holding prices constant, $E\pi_n = E\pi_q = E\pi_z = 0$. Substitute (A7) and (A8) into (A10) to obtain

$$(A11) \quad \Omega(EN/EI) = (1+k)\{\eta_n[1 + k\check{s}_n b_n \sigma_{nq} + (1-k\check{q})s_q(1-b_q)\sigma_q] \\ + \eta_q[k_q s_q(1-b_q)\sigma_{nq} + (1-k_n)s_n b_n \sigma_n]\};$$

$$\Omega(EQ/EI) = (1+k)\{\eta_q[1 + k\check{q}s_q b_n \sigma_{nq} + (1-k\check{n})s_n(1-b_n)\sigma_n] \\ + \eta_n[k\check{n}s_n(1-b_n)\sigma_{nq} + (1-k\check{q})s_q b_n \sigma_q]\};$$

$$\Omega = \Omega_{11}\Omega_{22} - \Omega_{12}\Omega_{21};$$

$$\Omega_{11} = [1 + \eta_n k\check{n}s_n + (1-k\check{n})s_n(1-b_n)\sigma_n + k\check{q}s_q b_n \sigma_{nq}];$$

$$\Omega_{12} = [\eta_n k\check{q}s_q - (1-k\check{n})s_n b_q \sigma_n - k\check{q}s_q(1-b_q)\sigma_{nq}];$$

$$\Omega_{21} = [\eta_q k\check{n}s_n - (1-k\check{q})s_q b_n \sigma_q - k\check{n}s_n(1-b_n)\sigma_{nq}];$$

$$\Omega_{22} = [1 + \eta_q k\check{q}s_q + (1-k\check{q})s_q(1-b_q)\sigma_q + k\check{n}s_n b_q \sigma_{nq}],$$

where Ω must be positive by the second-order conditions. An interesting issue is the sign of $(EN/EI - EQ/EI)$. By assuming that all the Allen partial elasticities of substitution are identical, $\sigma_{nq} = \sigma_{nz} = \sigma_{qz} = \sigma$, and that the family business production function has the form $F(NQ)$ implying $b_n = b_q = b$, we can derive the following expression using (A11):

$$(A12) \quad \Omega(EQ/EI - EN/EI) = (1+k)[1+(1-2b)(1-k)b](\eta_q - \eta_n)$$

A necessary but not sufficient condition for $EQ/EI - EN/EI$ to have a sign that is opposite to $\eta_q - \eta_n$ is to have $(1-2b) < 0$. This requires that F be an increasing returns to scale production function, which is contrary to our assumption.

The income compensated price elasticities of demand for N and Q can be derived in essentially the same manner. Substitute (A7) and (A8) into (A11) to obtain

$$(A13) \quad \Omega Ei/E\pi_i = [k\check{j}s_j(1-b_j)\sigma_{ij} - k\check{i}s_i\eta_i]k\check{i}(1+s_i)\sigma_{ij} \\ - [1+k\check{j}s_j\eta_j + (1-k\check{j})s_j(1-b_j)\sigma_j](1-k\check{i})(1+s_i)\sigma_i,$$

$$\begin{aligned} \Omega E_i/E\pi_j^* &= [1+k_j^*s_j\eta_j+k_j^*s_i b_j\sigma_{ij}]k_j^*(1+s_j)\sigma_{ij} \\ &\quad - [(1-k_j^*)s_i b_j\sigma_i-k_j^*s_j\eta_i](1-k_j^*)(1+s_j)\sigma_j, \end{aligned}$$

($i \neq j = N, Q$).

The compensated own price elasticities are negative because one can eliminate the only positive term on the right hand side of (A13) by noting that

$$\begin{aligned} k_j^*s_j(1-b_j)\sigma_{ij}k_j^*(1+s_i)\sigma_{ij} \\ < (1-k_j^*)s_j(1-b_j)\sigma_j(1-k_j^*)(1+s_i)\sigma_i. \end{aligned}$$

The signs of the compensated cross price elasticities are ambiguous. The only negative term on the right cannot be eliminated because

$$k_j^*s_i b_j\sigma_{ij}k_j^*(1+s_j)\sigma_{ij} < (1-k_j^*)s_i b_j\sigma_i(1-k_j^*)(1+s_j)\sigma_j.$$

An important effect to evaluate is the response of N and Q to changes in W_f . An increase in W_f increases the marginal shadow costs, π_i 's, for all three commodities according to

$$(A14) \quad E\pi_i = a_i E W_f; \quad a_i = (W_f t_{fi})/\pi_i; \quad (i = N, Q, Z),$$

where a_i is the relative cost share of wife's time in the production of commodity i and is a measure of the time intensity of commodity i in cost terms. Notice that the appropriate measure of time intensity of commodity i in price terms is given by $a_i^* = (W_f t_{fi})/\pi_i^*$. For Z the two measures of time intensity are equal, but for N and Q we have $a_i < a_i^*$. Hence, even when the time intensities in cost terms are equal for all commodities, the time intensities for N and Q in price terms will be greater than Z . An income compensated increase in wife's wage rate will therefore decrease the demand for N and Q relative to Z . The effect will be strengthened if the usual presumption that Z is less cost time intensive than N and Q is assumed.

The marginal value products of children in the family business with respect to N and Q , PF_N and PF_Q , will not remain constant when the demand for N and Q decline relative to Z . The relative

shadow price between N and Q may change even though the relative marginal shadow cost between N and Q remains constant. The effect on the relative price between N and Q will depend on s_i ($=PF_i/\pi_i^*$) and on the relative magnitudes of the change in PF_n and PF_q . If $s_n < s_q$, a rise in W_f is more likely to increase π_n/π_q , even though the cost time intensities for N and Q are equal. The effect would be strengthened if N is more cost time intensive than Q.

Using (A10) and (A13) we can derive the effect of an equal percentage change in all the π_i s on N and Q. Define $E\pi^0 = E\pi_n = E\pi_q = E\pi_z$.

$$(A14) \quad \Omega \frac{EN}{E\pi^0} = [-(1-k_n^*)(1+s_n)\sigma_n + k_q^*(1+s_q)\sigma_{nq} + k_z^*\sigma_{nz}]\Omega_{22} \\ - [k_n^*(1+s_n)\sigma_{nq} - (1-k_q^*)(1+s_q)\sigma_q + k_z^*\sigma_{qz}]\Omega_{12}; \\ -\Omega \frac{EQ}{E\pi^0} = [k_n^*(1+s_n)\sigma_{nq} - (1-k_q^*)(1+s_q)\sigma_q + k_z^*\sigma_{qz}]\Omega_{21} \\ - [-(1-k_n^*)(1+s_n)]\sigma_n + k_q^*(1+s_q)\sigma_{nq} + k_z^*\sigma_{nz}]\Omega_{11},$$

The sign of $\Omega[EN/E\pi^0 - EQ/E\pi^0]$ can be evaluated by simplifying (A14). Assume that $\sigma_{nq} = \sigma_{nz} = \sigma_{qz} = \sigma$ and collect terms using the well-known proposition that

$$(A15) \quad 1 = k_n^* + k_q^* + k_z^* \\ = [k_n^*(1+s_n) + k_q^*(1+s_q) + k_z^*](1+k),$$

to yield

$$(A16) \quad \Omega[EN/E\pi^0 - EQ/E\pi^0] = \sigma[(k-s_n)(\Omega_{22}-\Omega_{21}) + (k-s_q)(\Omega_{11}-\Omega_{12})]$$

In evaluating the sign of (A16) we make the plausible assumptions that $k-s_n < 0$ and $k-s_q < 0$. Since $k = k_n^*s_n + k_q^*s_q$, these assumptions require the values of s_n and s_q to be bounded by

$$k_q^*/(1-k_n^*) < s_n/s_q < (1-k_q^*)/k_n^*.$$

Inspecting (A16) reveals that all the terms in the expressions $(\Omega_{22}-\Omega_{21})$ and $(\Omega_{11}-\Omega_{12})$ are positive except for $(k_q^*s_q - k_n^*s_n)\eta_q$ and $(k_q^*s_q - k_n^*s_n)\eta_n$. A sufficient condition for (A16) to be negative is to have

$$(k-s_n)(k\check{s}_q-k\check{n}s_n)\eta_q - (k-s_q)(k\check{s}_q-k\check{n}s_n)\eta_n \leq 0.$$

This condition would be satisfied if the family business production function has the form $F(NQ)$

because this implies that

$$s_n/s_q = (PF'Q/\pi\check{n})/(PF'N/\pi\check{q}) = (\pi\check{q}Q/R)/(\pi\check{n}N/R) = k\check{q}/k\check{n},$$

so that $k\check{q}s_q = k\check{n}s_n$.

Appendix 2

Reduced Form Maximum Likelihood Logit Estimates of
Husband's and Wife's Choice of Economic Activity

	Husband Wage Employee	Wife Wage Employee	Wife Family Business	Wife Home Worker
Intercept	2.2410 (6.15)	-2.3018 (7.08)	-3.4658 (4.68)	-2.9438 (4.50)
Husband's age	-.0348 (4.09)	-.0272 (3.09)	.0077 (0.49)	-.0039 (0.29)
Wife's age	.0059 (0.61)	.0360 (3.71)	.0326 (1.81)	-.0118 (0.78)
Husband's schooling	.0386 (3.57)	.0028 (0.26)	-.0532 (2.70)	-.0346 (2.06)
Wife's schooling	.0063 (0.57)	.0771 (6.95)	-.0521 (2.52)	-.0017 (0.10)
Husband born in HK	-.5073 (4.47)	.0143 (0.15)	.1927 (0.87)	.1026 (0.60)
Wife born in HK	-.2450 (2.45)	-.1231 (1.35)	.1849 (0.94)	.3091 (1.99)
Husband Chinese	.2902 (1.07)	-.1080 (0.46)	-.9569 (2.05)	.4886 (0.90)
Wife Chinese	-.1343 (0.53)	.7234 (3.06)	.9034 (1.54)	.5551 (1.13)
Subsidized renter	1.1091 (10.84)	-.1140 (1.24)	-.6440 (3.39)	.8092 (0.06)
Homeowner	-.3986 (4.00)	-.2688 (2.55)	.4194 (2.25)	-.0795 (0.38)
Extended family	.0068 (0.06)	.4538 (0.47)	-.0999 (0.48)	-.2139 (1.20)
-2 log likelihood	3780.45		7373.34	

Cases

4128

4128

Note: Absolute asymptotic t-values in parentheses.

Appendix 3

OLS Estimates of Husband's and Wife's Log Monthly Earnings
by Economic Activity with Correction for Selectivity Bias

	Husband Wage Employee	Husband Family Business	Wife Wage Employee	Wife Family Business	Wife Home Worker
Intercept	6.2705 (32.25)	7.7441 (14.49)	4.4611 (9.35)	5.8263 (2.49)	4.9825 (3.49)
Schooling	-.0009 (0.09)	-.0250 (0.88)	-.0028 (0.13)	-.1725 (1.50)	.0514 (2.60)
Schooling ²	.0025 (10.90)	.0033 (5.81)	.0049 (0.83)	.0127 (3.88)	-.0029 (-1.15)
Schooling'Age .	0001 (0.53)	.0005 (1.00)	-.0004 (0.87)	.0026 (1.05)	.00003 (0.01)
Age	.0388 (4.86)	.0235 (1.18)	.0588 (3.05)	-.0339 (0.31)	-.0665 (1.01)
Age ²	-.0005 (5.86)	-.0005 (2.30)	-.0007 (2.80)	.0003 (0.20)	.0009 (1.04)
Born in HK	-.1729 (8.30)	-.1511 (2.32)	-.2133 (5.63)	-.0761 (0.44)	-.0181 (0.17)
Chinese	-.1903 (4.62)	-.2381 (1.69)	.4927 (5.55)	1.6956 (2.37)	.4927 (1.89)
$\hat{\alpha}$	-.1660 (3.07)	-.5664 (6.80)	-.1395 (0.97)	-.3866 (1.12)	-.4980 (2.24)
Adj. R ²	.3310	.2530	.4551	.1544	.3600
Cases	3249	879	980	207	309

Note: 1. Absolute t-values in parentheses.
2. $\hat{\alpha}$ is the selection bias control variable constructed from the estimates in Appendix 2

Family Business and the Quantity and Quality of Children
School of Economics and Finance, The University of Hong Kong
using the method proposed by Lee (1983).

Table 1

Fertility and Child Schooling by
Husband's and Wife's Economic Activity

		Husband's economic activity		
		Family Wage business	employee	Total
Wife's economic activity				
Not working	N	3.5564	3.0796	3.1809
	Cases	559	2073	2632
	Q	1.0206	1.0203	1.0204
	Cases	531	1933	2464
Family business	N	4.2152	4.1429	4.1973
	Cases	158	49	207
	Q	0.9965	0.9851	0.9938
	Cases	150	47	197
Home worker	N	3.6410	3.5741	3.5828
	Cases	39	270	309
	Q	1.0035	1.0070	1.0066
	Cases	40	263	303
Wage employee	N	3.2358	2.3384	2.4510
	Cases	123	857	980
	Q	1.0229	1.0140	1.0151
	Cases	105	699	804
Total	N	3.6337	2.9412	3.0887
	Cases	879	3249	4128
	Q	1.0158	1.0170	1.0167
	Cases	826	2942	3768

Note: N is the number of children and Q is the child schooling index. Q applies to families with at least one child.

Table 2
Means and Standard Deviations of Variables

	Families with at least one child		All families	
	Mean	Std. dev.	Mean	Std. dev.
Number of children	3.3291	1.8838	3.0315	2.0332
log(no. of children + 1)	1.3719	0.4361	1.2492	0.5713
Child schooling index	1.0183	0.1365	1.0167	0.1307
Husband's age	41.7218	8.7008	40.9207	9.1135
Wife's age	36.4402	7.9099	35.7386	8.2082
Husband's schooling	9.1433	5.0082	9.3275	5.0082
Wife's predicted log home work earnings	4.1721	0.1272	4.1734	0.1285
Wife's predicted log business earnings	6.9726	0.7641	7.0025	0.7946
Wife's predicted log employee earnings	6.2423	0.3528	6.2611	0.3649
Husband's predicted log business earnings	7.7791	0.4053	7.8144	0.4098
Husband's predicted log employee earnings	6.9428	0.3296	6.9555	0.3336
Extended family (dummy variable)	0.1751	0.3801	0.1690	0.3748
Subsidized renter (dummy variable)	0.4620	0.4986	0.4358	0.4959
Homeowner (dummy variable)	0.2346	0.4238	0.2289	0.4201
Cases		3768		4128

Table 3

Nonlinear GLS Estimates of Fertility and Child Schooling

	Families with at least one child		All families
	Fertility	Child Schooling	Fertility
Intercept	1.2878 (2.52)	.9150 (5.13)	.0436 (0.07)
Husband's schooling	-.0120 (6.06)	.0027 (3.72)	-.0143 (5.87)
Wife's schooling	-.0193 (6.78)	.0037 (3.60)	-.0245 (7.19)
Wife's predicted log home work earnings	.0914 (1.52)	.0544 (2.57)	.2253 (3.19)
Wife's predicted log business earnings	.0837 (3.27)	.0045 (0.50)	.0816 (2.67)
Wife's predicted log employee earnings	-.2171 (3.04)	-.0542 (2.20)	-.1870 (2.16)
Husband's predicted log business earnings	.1212 (8.36)	.0104 (1.97)	.1877 (10.93)
Husband's predicted log employee earnings	-.0600 (3.48)	.0051 (0.81)	-.0790 (3.75)
Wife's age - 15 (nonlinear term)	.2473 (45.76)	-	.2101 (48.56)
R ²	.2975	.0341	.3475
Cases	3768	3768	4128

Note: Absolute t-values in parentheses.